STATISTICAL METHODS FOR CHANGE DETECTION

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Summary

Handling parameterized (or parametric) models for monitoring industrial processes is a natural approach to fault detection and isolation. A key feature of the statistical approach is its ability to handle noises and uncertainties. Modeling faults such as deviations in the parameter vector with respect to (w.r.t.) its nominal value calls for the use of statistical change detection and isolation methods. The purpose of this article is to introduce key concepts for the design of such algorithms.

1. Introduction

1.1 Motivations for Change Detection

Many monitoring problems can be stated as the problem of detecting and isolating a change in the parameters of a static or dynamic stochastic system. The use of a model of the monitored system is reasonable, since many industrial processes rely on physical principles, which write in terms of (differential) equations, providing us with (dynamical) models. Moreover, the use of (physical) parameters is mandatory when isolation and diagnosis are sought.

In the sequel, we equally use the words deviation, change, failure, fault, damage, malfunction, considering that all these events are reflected by a change in the parameter vector of a model of the system.

The change detection framework and methodology is one way to approach the analysis of nonstationary phenomena. Statistical decision tools for detecting and estimating changes are useful for different purposes:

1. Automatic segmentation of signals as a first step in recognition-oriented signal processing;
2. Gain updating in adaptive identification algorithms for improving their tracking ability;
3. Quality control
4. Monitoring complex structures and industrial processes (fault detection and diagnosis), for fatigue prevention, aided control and condition-based maintenance.

Even though this chapter focuses on the use of change detection for fault detection and isolation, the same methodology and tools apply to the other problems as well.

1.2 Motivations for Statistical Methods

It has been widely acknowledged that the FDI (fault detection and isolation) problem can be split into two steps: *generation of residuals*, which are ideally close to zero under no-fault conditions, minimally sensitive to noises and disturbances, and maximally sensitive to faults; and *residual evaluation*, namely design of decision rules based on these residuals.

The basic statistical approach to residual generation consists in deriving sufficient statistics, namely transformations of the measurements which capture the entire information about the fault contained in the original data. Residual evaluation is typically answerable to statistical methods, which are basically aimed at deciding if a residual discrepancy from zero is significant.

The main advantage of the statistical approach is its ability to asses the level of significance of discrepancies with respect to uncertainties. The accuracy of parameter estimates provides us with the relative size of the estimation error w.r.t the noises on the system measurements.

Similarly, statistical tests described in the following can tell us if the relative size of the parameter discrepancy in the monitored system w.r.t to the accuracy of the reference parameter value is significant or not.

However, an essential issue when dealing with component faults is that the prediction error is not the relevant function of the model parameter and the measured data to be computed for stating this significance. The gradient of the squared prediction error w.r.t the parameter, or any other parameter estimating function, should be used instead.

1.3 Three Types of Change Detection Problems

From now on, we assume that we are given a reference value θ_0 of the model parameter. Generally, such a reference parameter is identified with data from the fault-free system. If, as it is often the case in practice, the monitored system is subject to other types of non-stationarities than the parameter deviations of interest, the reference value θ_0 should be identified using long data samples containing as many of these undesirable changes as possible. This holds true for changes in the functioning modes of a machine, non-stationarities in the environment of a process, etc.

The detection problem may be solved on the basis of data samples of smaller size. Depending on the relative time constants of the process to be monitored, on the sampling of the data, and on the size, speed and rate of the deviations to be detected, three types of detection problems may occur in practice, when processing real data, on-board or otherwise.

- *Model validation:* Given, on the one hand, a reference value θ_0 of the model parameter and, on the other hand, a new data sample, the problem is to decide whether the new data are still well described by this parameter value or not. Of course, this problem may be stated either off-line (fixed sample size N) or on-line (varying sample size n). A fixed-size sliding window may be useful.

- *Off-line change detection:* Given a data sample of size N, the problem is to decide whether, somewhere in this sample, a change in the parameter has occurred, from the value θ_0 to the value θ_1, at an unknown time instant υ.

- *On-line change detection:* At every time instant n, the problem is to decide whether, before this instant, a change in the parameter has occurred, from the value θ_0 to the value θ_1, at an unknown time instant υ.

Of course, the most difficult problem is the third one, because in this problem the amount of information in the data about the new parameter value θ_1 is the lowest. Also, the criteria for designing the detection algorithms and analyzing their performances depend on the detection problem. These are: mean time between false alarms, probability of wrong detection, mean delay to detection, probability of non-detection, accuracy of the estimates of the fault onset time and of the magnitude of the change.

Even though the decision functions for solving these three problems are not the same, they all can be viewed as different implementations of the same primary residual. This chapter puts some emphasis on the model validation problem, which is the simplest. Model validation may be a relevant issue for on-board processing: for example, batch processing is appropriate for on-board monitoring of aging.

2. Foundations: Detection

The key statistical tools for fault/change detection rely on hypotheses testing and ratios of likelihoods, or on approximations of those ratios. One major approximation, with the assumption of small change, is the gradient of the likelihood function; this gives rise to the so-called local approach to the design of detection algorithms. This approach can be extended to other parameter estimating functions than the likelihood gradient.

2.1. Likelihood Ratio and CUSUM Tests

The key detection tools are first introduced for hypotheses testing, then for on-line change detection, distinguishing between independent and dependent observed data.

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Biographical Sketch

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